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Broadening the Higgs Boson with Right-Handed Neutrinos and a Higher Dimension Operator at the Electroweak Scale

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Abstract

The existence of certain TeV suppressed higher-dimension operators may open up new decay channels for the Higgs boson to decay into lighter right-handed neutrinos. These channels may dominate over all other channels if the Higgs boson is light. For a Higgs boson mass larger than $2m_W$ the new decays are subdominant yet still of interest. The right-handed neutrinos have macroscopic decay lengths and decay mostly into final states containing leptons and quarks. A distinguishing collider signature of this scenario is a pair of displaced vertices violating lepton number. A general operator analysis is performed using the minimal flavor violation hypothesis to illustrate that these novel decay processes can occur while remaining consistent with experimental constraints on lepton number violating processes. In this context the question of whether these new decay modes dominate is found to depend crucially on the approximate flavor symmetries of the right-handed neutrinos.

1 Motivation

Neutrino interactions with other Standard Model particles are well-described, forming a cornerstone of the Standard Model itself. But the origin of their masses remain unknown. If their masses are generated by a local quantum field theory, then other degrees of freedom must exist. These particles or "right-handed neutrinos" are either the missing Dirac partners of the neutrinos, or are much heavier than the O(eV) mass scale of the neutrinos and create a "see-saw" mechanism. Which of these scenarios is realized in Nature is dependent on the unknown scale of the Majorana mass parameters of the right-handed neutrinos.

The existence of right-handed neutrinos may have other physical consequences, depending on the size of their Majorana masses. Right-handed neutrinos with Majorana masses violate overall lepton number, which may have consequences for the origin of the observed baryon asymmetry. Leptogenesis can occur from the out of equilibrium decay of a right-handed neutrino with mass larger than the TeV scale [1]. Interestingly, right-handed neutrinos with masses below the electroweak scale may also lead to baryogenesis [2].

But if right-handed neutrinos exist, where did their mass come from? The Majorana mass parameters are not protected by the gauge invariance of the Standard Model, so an understanding of the origin of their mass scale requires additional physics. The see-saw mechanism with order unity Yukawa couplings prefers a large scale, of order 10^{13-14} GeV. But in this case a new, intermediate scale must be postulated in addition to the four mass scales already observed in Nature. On the other hand, such a large scale might occur naturally within the context of a Grand Unified Theory.

Here I explore the consequences of assuming that the Majorana neutrino mass scale is generated at the electroweak scale [1]. To then obtain the correct mass scale for the left-handed neutrinos from the "see-saw" mechanism, the neutrino Yukawa couplings must be tiny, but not unreasonably small, since they would be comparable to the electron Yukawa coupling. It might be natural for Majorana masses much lighter than the Planck or Grand Unified scales to occur in specific Randall-Sundrum type models [7] or their CFT dual descriptions by the AdS/CFT correpsondance [8]. But as the intent of this paper is to be as model-independent as possible, I will instead assume that it is possible to engineer electroweak scale Majorana masses and use effective field theory to describe the low-energy theory of the Higgs boson and the right-handed and left-handed (electroweak) neutrinos. I will return to question of model-building in the concluding section and provide a few additional comments.

With the assumption of a common dynamics generating both the Higgs and right-handed neutrino mass scales, one may then expect strong interactions between these particles, in the form of higher dimension operators. However since generic flavor-violating higher dimension operators involving Standard Model fields and suppressed only by the TeV are excluded, I will use throughout the minimal flavor violation hypothesis [9, 10, 11] in

[1] For previous work on the phenomenology of electroweak scale right-handed neutrinos, see [3, 4, 5, 6]. None of these authors consider the effects of TeV-scale suppressed higher dimension operators.

order to suppress these operators. The purpose of this paper is to show that the existence of operators involving the Higgs boson and the right-handed neutrinos can significantly modify the phenomenology of the Higgs boson by opening a new channel for it to decay into right-handed neutrinos. I show that the right-handed neutrinos are long-lived and generically have macroscopic decay lengths. For reasonable values of parameters their decay lengths are anywhere from fractions of a millimeter to tens of metres or longer if one of the left-handed neutrinos is extremely light or massless. As they decay predominantly into a quark pair and a charged lepton, a signature for this scenario at a collider would be the observation of two highly displaced vertices, each producing particles of this type. Further, by studying these decays all the CP-preserving parameters of the right-handed and left-handed neutrinos interactions could be measured, at least in principle.

A number of scenarios for new physics at the electroweak scale predict long-lived particles with striking collider features. Displaced vertices due to long-lived neutral particles or kinks appearing in charged tracks are predicted to occur in models of low energy gauge mediation [12]. More recently models with a hidden sector super-Yang Mills coupled weakly through a Z' or by mass mixing with the Higgs boson can produce dramatic signatures with displaced jets or leptons and events with high multiplicity [13]. A distinguishing feature of the Higgs boson decay described here is the presence of two displaced vertices where the particles produced at each secondary vertex violate overall lepton number.

That new light states or operators at the electroweak scale can drastically modify Higgs boson physics has also been recently emphasized. Larger neutrino couplings occur in a model with nearly degenerate right-handed neutrino masses and vanishing tree-level active neutrino masses, that are then generated radiatively at one-loop [3]. Decays of the Higgs boson into a right-handed and left-handed neutrino may then dominate over decays to bottom quarks if the right-handed neutrinos are heavy enough. Models of supersymmetry having pseudoscalars lighter than the neutral Higgs scalar may have exotic decay processes for the Higgs boson that can significantly affect limits and searches [14]. Supersymmetry without R-parity can have striking new signatures of the Higgs boson [15]. Two common features between that reference and the work presented here is that the Higgs boson decays into a 6-body final state and may be discovered through displaced vertices, although the signatures differ.

Interesting phenomena can also occur without supersymmetry. Adding to the Standard Model higher dimension operators involving only Standard Model fields can modify the Higgs boson production cross-section and branching fractions [16]. Such an effect can occur in models with additional colored scalars coupled to top quarks [17].

The outline of the paper is the following. Section 2 discusses the new decay of the Higgs boson into right-handed neutrinos. Section 3 then discusses various naturalness issues that arise in connection with the relevant higher dimension operator. Section 4 discusses predictions for the coefficients of the new operator within the framework of minimal flavor violation [9, 10, 11]. It is found that the predicted size of the higher dimension operators depends crucially on the approximate flavor symmetries of the right-handed neutrinos. How

this affects the branching ratio for the Higgs boson to decay into right-handed neutrinos is then discussed. Section 5 computes the lifetime of the right-handed neutrinos assuming minimal flavor violation and discusses its dependence on neutrino mass parameters and mixing angles. I conclude in Section 6 with some comments on model-building issues and summarize results.

2 Higgs Boson Decay

The renormalizable Lagrangian describing interactions between the Higgs doublet H $(\mathbf{1},\mathbf{2})_{-1/2}$, the lepton $SU(2)_W$ doublets L_i $(\mathbf{1},\mathbf{2})_{-1/2}$, and three right-handed neutrinos N_I $(\mathbf{1},\mathbf{1})_0$ is given by

$$\mathcal{L}_{\mathcal{R}} = \frac{1}{2} m_R N N + \lambda_\nu \tilde{H} N L + \lambda_l H L e^c \tag{1}$$

where flavor indices have been suppressed and $\tilde{H} \equiv i\tau_2 H^*$ where H has a vacuum expectation value (vev) $\langle H \rangle = v/\sqrt{2}$ and $v \simeq 247$ GeV. Two-component notation is used throughout this note. We can choose a basis where the Majorana mass matrix m_R is diagonal and real with elements M_I. In general they will be non-universal. It will also be convenient to define the 3×3 Dirac neutrino mass $m_D \equiv \lambda_\nu v/\sqrt{2}$. The standard see-saw mechanism introduces mass mixing between the right-handed and left-handed neutrinos which leads to the active neutrino mass matrix

$$m_L = \frac{1}{2}\lambda^T_\nu m^{-1}_R \lambda_\nu v^2 = m^T_D m^{-1}_R m_D \ . \tag{2}$$

This is diagonalized by the PMNS matrix U_{PMNS} [18] to obtain the physical masses m_I of the active neutrinos. At leading order in the Dirac masses the mass mixing between the left-handed neutrinos ν_I and right-handed neutrinos N_J is given by

$$V_{IJ} = [m^T_D m^{-1}_R]_{IJ} = [m^T_D]_{IJ} M^{-1}_J \tag{3}$$

and are important for the phenomenology of the right-handed neutrinos. For generic Dirac and Majorana neutrino masses no simple relation exists between the physical masses, left-right mixing angles and the PMNS matrix. An estimate for the neutrino couplings is

$$f_I \simeq 7\times 10^{-7} \left(\frac{m_I}{0.5\text{eV}}\right)^{1/2} \left(\frac{M}{30\text{GeV}}\right)^{1/2} \ . \tag{4}$$

where $\lambda_\nu = U_R f U_L$ has been expressed in terms of two unitary matrices $U_{L/R}$ and a diagonal matrix f with elements f_I. In general $U_L \neq U_{PMNS}$. Similarly, an approximate relation for the left-right mixing angles is

$$V_{IJ} \simeq \sqrt{\frac{m_J}{M}}[U_{PMNS}]_{JI} = 4\times 10^{-6}\sqrt{\left(\frac{m_J}{0.5\text{eV}}\right)\left(\frac{30\text{GeV}}{M}\right)}[U_{PMNS}]_{JI} \tag{5}$$

which is valid for approximately universal right-handed neutrino masses $M_I \simeq M$ and $U_R \simeq 1$. I note that these formulae for the masses and mixing angles are exact in the limit of universal Majorana masses and no CP violation in the Dirac masses [11]. For these fiducial values of the parameters no limits exist from the neutrinoless double β decay experiments or collider searches [5] because the mixing angles are too tiny. No limits from cosmology exist either since the right-handed neutrinos decay before big bang nucleosynthesis if $M_I \gtrsim O(\text{GeV})$, which will be assumed throughout (see Section 5 for the decay length of the right-handed neutrinos).

If a right-handed neutrino is lighter than the Higgs boson, $M_I < m_h$, where m_h is the mass of the Higgs boson, then in principle there may be new decay channels

$$h \to N_I + X \tag{6}$$

where X may be a Standard Model particle or another right-handed neutrino (in the latter case $M_I + M_J < m_h$). For instance, from the neutrino coupling one has $h \to N_I \nu_L$. This decay is irrelevant, however, for practical purposes since the rate is too small.

But if it is assumed that at the TeV scale there are new dynamics responsible for generating both the Higgs boson mass and the right-handed neutrino masses, then higher-dimension operators involving the two particles should exist and be suppressed by the TeV scale. These can be a source of new and *relevant* decay processes. Consider then

$$\delta\mathcal{L}_{eff} = \sum_i \frac{c_i^{(5)} \cdot}{\Lambda} \mathcal{O}_i^{(5)} + \sum_i \frac{c_i^{(6)} \cdot}{\Lambda^2} \mathcal{O}_i^{(6)} + \cdots + \text{h.c.} \tag{7}$$

where $\Lambda \simeq \mathcal{O}(\text{TeV})$. Only dimension 5 operators are considered here, with dimension 6 operators discussed elsewhere [19]. The central dot '$\cdot$' denotes a contraction of flavor indices.

At dimension 5 there are several operators involving right-handed neutrinos. However it is shown below that constraints from the observed scale of the left-handed neutrino masses implies that only one of them can be relevant. It is

$$\mathcal{O}_1^{(5)} = H^\dagger H N N \tag{8}$$

where the flavor dependence is suppressed. *The important point is that this operator is not necessarily suppressed by any small Yukawa couplings.* After electroweak symmetry breaking the only effect of this operator at tree-level is to shift the masses of the right-handed neutrinos. Constraints on this operator are therefore weak (see below).

This operator, however, can have a significant effect on the Higgs boson. For if

$$M_I + M_J < m_h \ , \tag{9}$$

the decay

$$h \to N_I N_J \tag{10}$$

can occur. For instance, if only a single flavor is lighter than the Higgs boson, the decay rate is

$$\Gamma(h \to N_I N_I) \;\; = \;\; \frac{v^2}{4\pi\Lambda^2} m_h \beta_I \left((\mathrm{Re} c_1^{(5)})^2 \beta_I^2 + (\mathrm{Im} c_1^{(5)})^2 \right) \tag{11}$$

where only half the phase space has been integrated over, $c_1^{(5)}/\Lambda$ is the coefficient of (8), and $\beta_I \equiv (1 - 4M_I^2/m_h^2)^{1/2}$ is the velocity of the right-handed neutrino.

The dependence of the decay rate on β may be understood from the following comments. The uninterested reader may skip this paragraph, since this particular dependence is only briefly referred to later in the next paragraph, and is not particularly crucial to any other discussion. Imagine a scattering experiment producing the two Majorana fermions only through an on-shell Higgs boson in the s-channel. The cross-section for this process is related to the decay rate into this channel, and in particular their dependence on the final state phase space are identical. Conservation of angular momentum, and when appropriate, conservation of CP in the scattering process then fixes the dependence of Γ on phase space. For example, note that the phase of $c_1^{(5)}$ is physical and cannot be rotated away. When $\mathrm{Im} c_1^{(5)} = 0$ the operator (8) conserves CP and the decay rate has the β^3 dependence typical for fermions. This dependence follows from the usual argument applied to Majorana fermions: a pair of Majorana fermions has an intrinsic CP parity of -1 [20], so conservation of CP and total angular momentum in the scattering process implies that the partial wave amplitude for the two fermions must be a relative p-wave state. If the phase of $c_1^{(5)}$ is non-vanishing, then CP is broken and the partial wave amplitude can have both p-wave and s-wave states while still conserving total angular momentum. The latter amplitude leads to only a β_I phase space suppression.

There is a large region of parameter space where this decay rate is larger than the rate for the Higgs boson to decay into bottom quarks, and, if kinematically allowed, not significantly smaller than the rate for the Higgs boson to decay into electroweak gauge bosons. For example, with $\mathrm{Im}(c_1^{(5)}) = 0$ and no sum over I,

$$\frac{\Gamma(h \to N_I N_I)}{\Gamma(h \to b\bar{b})} = \frac{2(c_1^{(5)})^2}{3} \frac{v^4}{m_b^2 \Lambda^2} \beta_I^3 \tag{12}$$

This ratio is larger than 1 for $\Lambda \lesssim 12 |c_1^{(5)}| \beta_I^{3/2}$ TeV . If all three right-handed neutrinos are lighter than the Higgs boson, then the total rate into these channels is larger than the rate into bottom quarks for $\Lambda \lesssim 20 |c_1^{(5)}| \beta_I^{3/2}$ TeV. If $\mathrm{Im}(c_1^{(5)}) \neq 0$ the operator violates CP and the region of parameter space where decays to right-handed neutrinos dominate over decays to bottom quarks becomes larger, simply because now the decay rate has less of a phase space suppression, as described above. The reason for the sensitivity to large values of Λ is because the bottom Yukawa coupling is small. For

$$m_h > 2 m_W \tag{13}$$

the Higgs boson can decay into a pair of W bosons with a large rate and if kinematically allowed, into a pair of Z gauge bosons with a branching ratio of approximately 1/3. One finds that with $\mathrm{Im}(c_1^{(5)}) = 0$ and no sum over I,

$$\frac{\Gamma(h \to N_I N_I)}{\Gamma(h \to WW)} = \frac{4(c_1^{(5)})^2 v^4}{m_h^2 \Lambda^2} \frac{\beta_I^3}{\beta_W} \frac{1}{f(\beta_W)} \tag{14}$$

where $f(\beta_W) = 3/4 - \beta_W^2/2 + 3\beta_W^4/4$ [21] and β_W is the velocity of the W boson. Still, the decay of the Higgs boson into right-handed neutrinos is not insignificant. For example, with $\Lambda \simeq 2$ TeV, $c_1^{(5)} = 1$ and $\beta_I \simeq 1$, the branching ratio for a Higgs boson of mass 300 GeV to decay into a single right-handed neutrino flavor of mass 30 GeV is approximately 5%. Whether the decays of the Higgs boson into right-handed neutrinos are visible or not depends on the lifetime of the right-handed neutrino. That issue is discussed in Section 5.

It is now shown that all the other operators at $d = 5$ involving right-handed neutrinos and Higgs bosons are irrelevant for the decay of the Higgs boson. Aside from (8), there is only one more linearly independent operator involving the Higgs boson and a neutrino,

$$\mathcal{O}_2^{(5)} = L\tilde{H}L\tilde{H} \ . \tag{15}$$

After electroweak symmetry breaking this operator contributes to the left-handed neutrino masses, so its coefficient must be tiny, $c_2^{(5)} v^2/\Lambda \lesssim O(m_{\nu_L})$. Consequently, the decay of the Higgs boson into active neutrinos from this operator is irrelevant. In Section 4 it is seen that under the minimal flavor violation hypothesis this operator is naturally suppressed to easily satisfy the condition above. It is then consistent to assume that the dominant contribution to the active neutrino masses comes from mass mixing with the right-handed neutrinos.

Other operators involving the Higgs boson exist at dimension 5, but all of them can be reduced to (15) and dimension 4 operators by using the equations of motion. For instance,

$$\mathcal{O}_3^{(5)} \quad \equiv \quad -i(\partial^\mu \overline{N})\overline{\sigma}^\mu L\tilde{H} \to m_R N L\tilde{H} + (\tilde{H}L)\lambda_\nu^T (L\tilde{H}) \ , \tag{16}$$

where the equations of motion were used in the last step. As a result, this operator does not introduce any new dynamics. Still, its coefficients must be tiny enough to not generate too large of a neutrino mass. In particular, enough suppression occurs if its coefficients are less than or comparable to the neutrino couplings. Under the minimal flavor violation hypothesis it is seen that these coefficients are naturally suppressed to this level.

Even if the operators $\mathcal{O}_2^{(5)}$ and $\mathcal{O}_3^{(5)}$ are not present at tree-level, they will be generated at the loop-level through operator mixing with $\mathcal{O}_1^{(5)}$. This is because the overall lepton number symmetry $U(1)_{LN}$ is broken with both the neutrino couplings and $\mathcal{O}_1^{(5)}$ present. However, such mixing will always involve the neutrino couplings and be small enough to not generate too large of a neutrino mass. To understand this, it is useful to introduce

a different lepton number under which the right-handed neutrinos are neutral and both the charged leptons and left-handed neutrinos are charged. Thus the neutrino couplings and the operators $\mathcal{O}_2^{(5)}$ and $\mathcal{O}_3^{(5)}$ violate this symmetry, but the operator $\mathcal{O}_1^{(5)}$ preserves it. In the limit that $\lambda_\nu \to 0$ this lepton number symmetry is perturbatively exact, so inserting $\mathcal{O}_1^{(5)}$ into loops can only generate $\mathcal{O}_2^{(5)}$ and $\mathcal{O}_3^{(5)}$ with coefficients proportional to the neutrino couplings. Further, $\mathcal{O}_2^{(5)}$ violates this symmetry by two units, so in generating it from loops of Standard Model particles and insertions of $\mathcal{O}_1^{(5)}$ it will be proportional to at least two powers of the neutrino couplings. Likewise, in generating $\mathcal{O}_3^{(5)}$ from such loops its coefficient is always proportional to at least one power of the neutrino coupling. In particular, $\mathcal{O}_2^{(5)}$ is generated directly at two-loops, with $c_2^{(5)} \propto \lambda_\nu^T \lambda_\nu c_1^{(5)}$. It is also generated indirectly at one-loop, since $\mathcal{O}_3^{(5)}$ is generated at one-loop, with $c_3^{(5)} \propto c_1^{(5)} \lambda_\nu$. These operator mixings lead to corrections to the neutrino masses that are suppressed by loop factors and at least one power of m_R/Λ compared to the tree-level result.

As a result, no significant constraint can be applied to the operator $\mathcal{O}_1^{(5)}$.[2] Instead the challenge is to explain why the coefficients of $\mathcal{O}_2^{(5)}$ and $\mathcal{O}_3^{(5)}$ in the effective theory are small to begin with. The preceding arguments show why it is technically natural for them to be small, even if $\mathcal{O}_1^{(5)}$ is present. The minimal flavor violation hypothesis discussed below does provide a technically consistent framework in which this occurs.

3 Naturalness

The operator

$$\frac{c_1^{(5)}}{\Lambda} H^\dagger H N N \tag{17}$$

violates chirality, so it contributes to the mass of the right-handed neutrino at both tree and loop level. At tree level

$$\delta m_R = c_1^{(5)} \frac{v^2}{\Lambda} = 60 c_1^{(5)} \left(\frac{\text{TeV}}{\Lambda}\right) \text{GeV} . \tag{18}$$

There is also a one-loop diagram with an insertion of this operator. It has a quadratic divergence such that

$$\delta m_R \simeq 2 c_1^{(5)} \frac{\Lambda}{16\pi^2} . \tag{19}$$

Similarly, at one-loop

$$\delta m_h^2 \simeq \frac{1}{16\pi^2} \text{Tr}[c_1^{(5)} m_R] \Lambda . \tag{20}$$

[2]This statement assumes $c^{(5)} \lesssim O(16\pi^2)$ and that the loop momentum cutoff $\Lambda_{\text{loop}} \simeq \Lambda$. Constraints might conceivably occur for very light right-handed neutrino masses, but that possibility is not explored here since $M_I \gtrsim O(\text{GeV})$ is assumed throughout in order that the right-handed neutrinos decay before big bang nucleosynthesis.

If $c_1^{(5)} \sim O(1)$ then a right-handed neutrino with mass $M_I \simeq 30$ GeV requires $O(1)$ tuning for TeV$\lesssim \Lambda \lesssim 10$ TeV, and $m_h \simeq 100$ GeV is technically natural unless $\Lambda \gtrsim 10$ TeV or m_R is much larger than the range ($M_I \lesssim 150$ GeV) considered here.

Clearly, if $\Lambda \gtrsim O(10\ \text{TeV})$ then a symmetry would be required to protect the right-handed neutrino and Higgs boson masses. One such example is supersymmetry. Then this operator can be generalized to involve both Higgs superfields and would appear in the superpotential. It would then be technically natural for the Higgs boson and right-handed neutrino masses to be protected, even for large values of Λ. As discussed previously, for such large values of Λ decays of the Higgs boson into right-handed neutrinos may still be of phenomenological interest.

4 Minimal Flavor Violation

The higher dimension operators involving right-handed neutrinos and Standard Model leptons previously discussed can *a priori* have an arbitrary flavor structure and size. But as is well-known, higher dimension operators in the lepton and quark sector suppressed by only $\Lambda \simeq$ TeV -10 TeV are grossly excluded by a host of searches for flavor changing neutral currents and overall lepton number violating decays.

A predictive framework for the flavor structure of these operators is provided by the minimal flavor violation hypothesis [9, 10, 11]. This hypothesis postulates a flavor symmetry assumed to be broken by a minimal set of non-dynamical fields, whose vevs determine the renormalizable Yukawa couplings and masses that violate the flavor symmetry. Since a minimal field content is assumed, the flavor violation in higher dimension operators is completely determined by the now *irreducible* flavor violation appearing in the right-handed neutrino masses and the neutrino, charged lepton and quark Yukawa couplings. Without the assumption of a minimal field content breaking the flavor symmetries, unacceptably large flavor violating four fermion operators occur. In practice, the flavor properties of a higher dimension operator is determined by inserting and contracting appropriate powers and combinations of Yukawa couplings to make the operator formally invariant under the flavor group. Limits on operators in the quark sector are $5-10$ TeV [10], but weak in the lepton sector unless the neutrinos couplings are not much less than order unity [11][22].

It is important to determine what this principle implies for the size and flavor structure of the operator

$$(c_1^{(5)})_{IJ} H^\dagger H N_I N_J \ . \tag{21}$$

It is seen below that the size of its coefficients depends critically on the choice of the flavor group for the right-handed neutrinos. This has important physical consequences which are then discussed.

In addition one would like to determine whether the operators $\mathcal{O}_2^{(5)}$ and $\mathcal{O}_3^{(5)}$ are sufficiently suppressed such that their contribution to the neutrinos masses is always subdominant. In Section 2 it was argued that if these operators are initially absent, radiative

corrections involving $\mathcal{O}_1^{(5)}$ and the neutrino couplings will never generate large coefficients (in the sense used above) for these operators. However, a separate argument is needed to explain why they are initially small to begin with. It is seen below that this is always the case assuming minimal flavor violation.

To determine the flavor structure of the higher dimension operators using the minimal flavor violation hypothesis, the transformation properties of the particles and couplings are first defined. The flavor symmetry in the lepton sector is taken to be

$$G_N \times SU(3)_L \times SU(3)_{e^c} \times U(1) \tag{22}$$

where $U(1)$ is the usual overall lepton number acting on the Standard Model leptons. With right-handed neutrinos present there is an ambiguity over what flavor group to choose for the right-handed neutrinos, and what charge to assign them under the $U(1)$. In fact, since there is always an overall lepton number symmetry unless both the Majorana masses and the neutrino couplings are non-vanishing, there is a maximum of two such $U(1)$ symmetries.

Two possibilities are considered for the flavor group of the right-handed neutrinos:

$$G_N = SU(3) \times U(1)' \text{ or } SO(3) \ . \tag{23}$$

The former choice corresponds to the maximal flavor group, whereas the latter is chosen to allow for a large coupling for the operator (21), shown below. The fields transform under the flavor group $SU(3) \times SU(3)_L \times SU(3)_{e^c} \times U(1)' \times U(1)$ as

$$N \rightarrow (\mathbf{3},\mathbf{1},\mathbf{1})_{(\mathbf{1},\mathbf{0})} \tag{24}$$

$$L \rightarrow (\mathbf{1},\mathbf{3},\mathbf{1})_{(-\mathbf{1},\mathbf{1})} \tag{25}$$

$$e^c \rightarrow (\mathbf{1},\mathbf{1},\mathbf{3})_{(\mathbf{1},-\mathbf{1})} \ , \tag{26}$$

Thus $U(1)'$ is a lepton number acting on the right-handed neutrinos and Standard Model leptons and is broken only by the Majorana masses. $U(1)$ is a lepton number acting only on the Standard Model leptons and is only broken by the neutrino couplings. Then the masses and Yukawa couplings of the theory are promoted to spurions transforming under the flavor symmetry. Their representations are chosen in order that the Lagrangian is formally invariant under the flavor group. Again for $G_N = SU(3) \times U(1)'$,

$$\lambda_\nu \rightarrow (\overline{\mathbf{3}},\overline{\mathbf{3}},\mathbf{1})_{(\mathbf{0},-\mathbf{1})} \tag{27}$$

$$\lambda_l \rightarrow (\mathbf{1},\overline{\mathbf{3}},\overline{\mathbf{3}})_{(\mathbf{0},\mathbf{0})} \tag{28}$$

$$m_R \rightarrow (\overline{\mathbf{6}},\mathbf{1},\mathbf{1})_{(-\mathbf{2},\mathbf{0})} \ . \tag{29}$$

For $G_N = SO(3)$ there are several differences. First, the $\overline{\mathbf{3}}$'s of $SU(3)$ simply become $\mathbf{3}$'s of $SO(3)$. Next, the $U(1)$ charge assignments remain but there is no $U(1)'$ symmetry. Finally, a minimal field content is assumed throughout, implying that for $G_N = SO(3)$ $m_R \sim \mathbf{6}$ is real.

With these charge assignments a spurion analysis can now be done to estimate the size of the coefficents of the dimension 5 operators introduced in Section 2.

For either choice of G_N one finds the following. An operator that violates the $U(1)$ lepton number by n units is suppressed by n factors of the tiny neutrino couplings. In particular, the dangerous dimension 5 operators $\mathcal{O}_2^{(5)}$ and $\mathcal{O}_3^{(5)}$ are seen to appear with two and one neutrino couplings, which is enough to suppress their contributions to the neutrino masses. If $G_N = SO(3)$ such operators can also be made invariant under $SO(3)$ by appropriate contractions. If however $G_N = SU(3) \times U(1)'$, then additional suppressions occur in order to construct G_N invariants. For example, the coefficients of the dimension 5 operators $\mathcal{O}_2^{(5)}$ and $\mathcal{O}_3^{(5)}$ are at leading order $\lambda_\nu^T m_R^\dagger \lambda_\nu / \Lambda$ and $\lambda_\nu m_R^\dagger / \Lambda$ respectively and are sufficiently small.

It is now seen that the flavor structure of the operator (8) depends on the choice of the flavor group G_N. One finds

$$\begin{aligned} G_N = SU(3) \times U(1)' \quad &: \quad c_1^{(5)} \sim a_1 \frac{m_R}{\Lambda} + a_2 \frac{m_R \mathrm{Tr}[m_R^\dagger m_R]}{\Lambda^2} + \cdots \\ G_N = SO(3) \quad &: \quad c_1^{(5)} \sim \mathbf{1} + d_1 \frac{m_R}{\Lambda} + d_2 \frac{m_R \cdot m_R}{\Lambda^2} + \cdots + e_1 \lambda_\nu \lambda_\nu^\dagger + \cdots \end{aligned} \tag{30}$$

where $\cdots$ denotes higher powers in m_R and $\lambda_\nu \lambda_\nu^\dagger$. Comparing the expressions in (30), the only important difference between the two is that $\mathbf{1}$ is invariant under $SO(3)$, but not under $SU(3)$ or $U(1)'$. As we shall see shortly, this is a key difference that has important consequences for the decay rate of the Higgs boson into right-handed neutrinos.

Next the physical consequences of the choice of flavor group are determined. First note that if we neglect the $\lambda_\nu \lambda_\nu^\dagger \propto m_L$ contribution to $c_1^{(5)}$, then for either choice of flavor group the right-handed neutrino masses m_R and couplings $c_1^{(5)}$ are simultaneously diagonalizable. For $G_N = SO(3)$ this follows from the assumption that $m_R \sim \mathbf{6}$ is a real representation. As a result, the couplings $c_1^{(5)}$ are flavor-diagonal in the right-handed neutrino mass basis.

If $G_N = SO(3)$ the couplings $c_1^{(5)}$ are *flavor-diagonal, universal at leading order, and not suppressed by any Yukawa couplings.* It follows that

$$\frac{\mathrm{Br}(h \to N_I N_I)}{\mathrm{Br}(h \to N_J N_J)} = \frac{\beta_I^3}{\beta_J^3} \simeq 1 \tag{31}$$

up to small flavor-diagonal corrections of order m_R/Λ from the next-to-leading-order terms in the couplings $c_1^{(5)}$. β_I is the velocity of N_I and its appearance in the above ratio is simply from phase space. It is worth stressing that even if the right-handed neutrino masses are non-universal, the branching ratios of the Higgs boson into the right-handed neutrinos are approximately universal and equal to $1/3$ up to phase space corrections. The calculations from Section 2 of the Higgs boson decay rate into right-handed neutrinos do not need to be rescaled by any small coupling, and the conclusion that these decay channels dominate over

$h \to b\bar{b}$ for Λ up to 20 TeV still holds. Theoretically though, the challenge is to understand why $M_I \ll \Lambda$.

Similarly, if $G_N = SU(3)$ the couplings are *flavor-diagonal and suppressed by at least a factor of m_R/Λ but not by any Yukawa couplings.* This suppression has two effects. First, it eliminates the naturalness constraints discussed in Section 3. The other is that it suppresses the decay rate of $h \to N_I N_I$ by a predictable amount. In particular

$$\Gamma(h \to N_I N_I) \quad = \quad \frac{v^2}{4\pi\Lambda^2}\left(\frac{M_I}{\Lambda}\right)^2 m_h \beta_I^3 \tag{32}$$

where I have set $a_1 = 1$, and

$$\frac{\mathrm{Br}(h \to N_I N_I)}{\mathrm{Br}(h \to N_J N_J)} = \frac{M_I^2}{M_J^2}\frac{\beta_I^3}{\beta_J^3} \tag{33}$$

up to flavor-diagonal corrections of order m_R/Λ. In this case, the Higgs boson decays preferentially to the right-handed neutrino that is the heaviest. Still, even with this suppression these decays dominate over $h \to b\bar{b}$ up to $\Lambda \simeq 1$ TeV if three flavors of right-handed neutrinos of mass $M_I \simeq O(50\text{GeV})$ are lighter than the Higgs boson. For larger values of Λ these decays have a subdominant branching fraction. They are still interesting though, because they have a rich collider phenomenology and may still be an important channel in which to search for the Higgs boson. This scenario might be more natural theoretically, since an approximate $SU(3)$ symmetry is protecting the mass of the fermions.

5 Right-handed Neutrino Decays

I have discussed how the presence of a new operator at the TeV scale can introduce new decay modes of the Higgs boson into lighter right-handed neutrinos, and described the circumstances under which these new processes may be the dominant decay mode of the Higgs boson. In the previous section we have seen that whether that actually occurs or not depends critically on a few assumptions. In particular, on whether the Higgs boson is light, on the scale of the new operator, and key assumptions about the identity of the broken flavor symmetry of the right-handed neutrinos.

Whether the decays of the Higgs boson into right-handed neutrinos are visible or not depends on the lifetime of the right-handed neutrinos. It is seen below that in the minimal flavor violation hypothesis their decays modes are determined by their renormalizable couplings to the electroweak neutrinos and leptons, rather than through higher-dimension operators.

The dominant decay of a right-handed neutrinos is due to the gauge interactions with the electroweak gauge bosons it acquires through mass mixing with the left-handed neutrinos. At leading order a right-handed neutrino N_J acquires couplings to Wl_I and $Z\nu_I$

which are identical to those of a left-handed neutrino, except that they are suppressed by the mixing angles

$$V_{IJ} = [m_D^T]_{IJ} M_J^{-1} \ . \tag{34}$$

If the right-handed neutrino is heavier than the electroweak gauge bosons but lighter than the Higgs boson, it can decay as $N_J \rightarrow W^+ l_I^-$ and $N_J \rightarrow Z\nu_I$. Since it is a Majorana particle, decays to charge conjugated final states also occur. The rate for these decays is proportional to $|V_{IJ}|^2 M_J^3$.

If a right-handed neutrino is lighter than the electroweak gauge bosons, it decays through an off-shell gauge boson to a three-body final state. Its lifetime can be obtained by comparing it to the leptonic decay of the τ lepton, but after correcting for some additional differences described below. The total decay rate is [3]

$$\frac{\Gamma_{\text{total}}(N_I)}{\Gamma(\tau \rightarrow \mu \overline{\nu}_\mu \nu_\tau)} = 2 \times 9 \left(c_W + 0.40 c_Z\right) \frac{[m_D m_D^\dagger]_{II}}{M_I^2} \left(\frac{M_I}{m_\tau}\right)^5 \ . \tag{35}$$

The corrections are the following. The factor of "9" counts the number of decays available to the right-handed neutrino through charged current exchange, assuming it to be heavier than roughly few-10 GeV. The factor of "0.40" counts the neutral current contribution. It represents about 30% of the branching ratio, with the remaining 70% of the decays through the charged current. The factor of "2" is because the right-handed neutrino is a Majorana particle, so it can decay to both particle and anti-particle, e.g. $W^* l^-$ and $W^* l^+$, or $Z^*\nu$ and $Z^*\overline{\nu}$. Another correction is due to the finite momentum transfer in the electroweak gauge boson propagators. This effect is described by the factors c_W and c_W where

$$c_G(x_G, y_G) = 2 \int_0^1 dz z^2 (3-2z) \left((1-(1-z)x_G)^2 + y_G\right)^{-1} \tag{36}$$

where $x_G = M_I^2/m_G^2$, $y_G = \Gamma_G^2/m_G^2$, $c_G(0,0) = 1$ and each propagator has been approximated by the relativistic Breit-Wigner form. The non-vanishing momentum transfer enhances the decay rate by approximately 10% for m_R masses around 30GeV and by approximately 50% for masses around 50 GeV. This effect primarily affects the overall rate and is less important to the individual branching ratios.

The formula (36) is also valid when the right-handed neutrino is more massive than the electroweak gauge bosons such that the previously mentioned on-shell decays occur. In that case (35) gives the inclusive decay rate of a right-handed neutrino into any electroweak gauge boson and a charged lepton or a left-handed neutrino. In this case the correction from the momentum transfer is obviously important to include! It enhances the decay rate by approximately a factor of 40 for masses around 100 GeV, but eventually scales as M_I^{-2} for a large enough mass.

[3] An ≈ 2 error in an earlier version has been corrected.

An effect not included in the decay rate formula above is the quantum interference that occurs in the same flavor $l^+l^-\nu$ or $\nu\nu\overline{\nu}$ final states. Its largest significance is in affecting the branching ratio of these specific, subdominant decay channels and is presented elsewhere [19]. Using $c\tau_\tau = 87\mu m$ [23] and $BR(\rightarrow \mu\overline{\nu}_\mu\nu_\tau) = 0.174$ [23], (35) gives the following decay length for N_I,

$$c\tau_I = 0.90m \left(\frac{1.40}{c_W + 0.40c_Z}\right)\left(\frac{30\text{ GeV}}{M_I}\right)^3\left(\frac{(120\text{ keV})^2}{[m_D m_D^\dagger]_{II}}\right) . \qquad (37)$$

Care must be used in interpreting this formula, since the Dirac and Majorana masses are not completely independent because they must combine together to give the observed values of the active neutrino masses.

This expression is both model-independent and model-dependent. Up to this point no assumptions have been made about the elements of the Dirac mass matrix or the right-handed neutrino masses, so the result above is completely general. Yet the actual value of the decay length clearly depends on the flavor structure of the Dirac mass matrix. In particular, the matrix elements $[m_D m_D^\dagger]_{II}/M_I$ are not the same as the active neutrino mass masses. This is fortunate, since it presents an opportunity to measure a different set of neutrino parameters from those measured in neutrino oscillations.

The masses M_I describe 3 real parameters, and *a priori* the Dirac matrix m_D describes 18 real parameters. However, 3 of the phases in m_D can be removed by individual lepton number phase rotations on the left-handed neutrinos and charged leptons, leaving 15 parameters which I can think of as 6 mixing angles, 3 real Yukawa couplings and 6 phases. Including the three right-handed neutrino masses gives 18 parameters in total. Five constraints on combinations of these 18 parameters already exist from neutrino oscillation experiments. In principle all of these parameters could be measured through detailed studies of right-handed neutrino decays, since amplitudes for individual decays are proportional to the Dirac neutrino matrix elements. However, at tree-level these observables depend only on $|[m_D]_{IJ}|$ and are therefore insensitive to the 6 phases. So by studying tree-level processes only the 3 right-handed neutrino masses, 3 Yukawa couplings, and 6 mixing angles could be measured in principle.

In particular, the dominant decay is $h \rightarrow N_I N_I \rightarrow qqqql_J l_K$ which contains no missing energy. Since the secondary events are highly displaced, there should be no confusion about which jets to combine with which charged leptons. In principle a measurement of the mass of the right-handed neutrino and the Higgs boson is possible by combining the invariant momentum in each event. A subsequent measurement of a right-handed neutrino's lifetime from the spatial distribution of its decays measures $[m_D m_D^\dagger]_{II}$. More information is acquired by measuring the nine branching ratios $BR(N_I \rightarrow qq'l_J) \propto |[m_D]_{IJ}|^2$. Such measurements provide 6 additional independent constraints. In total, 12 independent constraints on the 18 parameters could in principle be obtained from studying right-handed neutrino decays at tree-level.

To say anything more precise about the decay length would require a model of the neutrino couplings and right-handed neutrino mass parameters. Specific predictions could be done within the context of such a model. Of interest would be the branching ratios and the mean and relative decay lengths of the three right-handed neutrinos.

The factor $[m_D m_D^\dagger]_{II}/M_I$ appearing in the decay length is not the active neutrino mass obtained by diagonalizing $m_D^T m_R^{-1} m_D$, but it is close. If I approximate $[m_D m_D^\dagger]_{II}/M_I \simeq m_I$, then

$$c\tau_I \simeq 0.90m \left(\frac{30 \text{ GeV}}{M_I}\right)^4 \left(\frac{0.48 \text{ eV}}{m_I}\right) \left(\frac{1.40}{c_W + 0.4c_Z}\right) \tag{38}$$

A few comments are in order. First, the decay lengths are macroscopic, since by inspection they range from $O(100\mu m)$ to $O(10m)$ for a range of parameters, and for these values are therefore visible at colliders. Next, the decay length is evidently extremely sensitive to M_I. Larger values of M_I have shorter decays lengths. For instance, if $M_I = 100$ GeV (which requires $m_h > 200$ GeV) and $m_I = 0.5$ eV then $c\tau_I \simeq 0.2mm$. Finally, if the active neutrino masses are hierarchical, then one would expect $M_I^4 c\tau_I$ to be hierarchical as well, since this quantity is approximately proportional to m_L^{-1}. One or two right-handed neutrinos may therefore escape the detector if the masses of the lightest two active neutrinos are small enough.

I have described decays of the right-handed neutrinos caused by its couplings to electroweak gauge bosons acquired through mass mixing with the left-handed neutrinos. However, additional decay channels occur through exchange of an off-shell Higgs boson, higher dimension operators or loop effects generated from its gauge couplings. It turns out that these processes are subdominant, but may be of interest in searching for the Higgs boson. Exchange of an off-shell Higgs boson causes a decay $N_I \rightarrow \nu_J b\bar{b}$ which is suppressed compared to the charged and neutral current decays by the tiny bottom Yukawa coupling. Similarly, the dimension 5 operator (8) with generic flavor couplings allows for the decay $N_I \rightarrow N_J h^* \rightarrow N_J b\bar{b}$ for N_J lighter than N_I [4]. However, using the minimal flavor violation hypothesis it was shown in Section 4 that the couplings of that higher dimension operator are diagonal in the same basis as the right-handed neutrino mass basis, up to flavor-violating corrections that are at best $O(\lambda_\nu^2)$ (see (30)). As result, this decay is highly suppressed. At dimension 5 there is one more operator that I have not yet introduced which is the magnetic moment operator

$$\frac{c_4^{(5)}}{\Lambda} \cdot N\sigma^{\rho\sigma} N B_{\rho\sigma} \tag{39}$$

and it involves only two right-handed neutrinos. It causes a heavier right-handed neutrino to decay into a lighter one, $N_I \rightarrow N_J + \gamma/Z$ for $I \neq J$. To estimate the size of this operator, first note that its coefficient must be anti-symmetric in flavor. Then in the context of minimal flavor violation with $G_R = SO(3)$, the leading order term is $c_4^{(5)} \simeq [\lambda_\nu \lambda_\nu^\dagger]_{AS}$ where

[4] The author thanks Scott Thomas for this observation.

"AS" denotes 'anti-symmetric part'. This vanishes unless the neutrino couplings violate CP. In that case the amplitude for this decay is of order $(\lambda_\nu)^2$. If $G_R = SU(3) \times U(1)'$ the leading order term cannot be $[m_R]_{AS}(\mathrm{Tr}[m_R m_R^\dagger]^q)^n/\Lambda^{n+q}$, since they vanish in the right-handed neutrino mass basis. The next order involves $\lambda_\nu \lambda_\nu^\dagger$ and some number of m_R's, but there does not appear to be any invariant term. Thus for either choice of G_R the amplitude for N_I decays from this operator are $O(\lambda_\nu^2)$ or smaller, which is much tinier than the amplitudes for the other right-handed neutrino decays already discussed which are of order λ_ν. Subdominant decays $N \to \nu + \gamma$ can occur from dimension 6 operators and also at also one-loop from electroweak interactions, but in both cases the branching ratio is tiny [19].

6 Discussion

In order for these new decays to occur at all requires that the right-handed neutrinos are lighter than the Higgs boson. But from a model building perspective, one may wonder why the right-handed neutrinos are not *heavier* than the scale Λ. A scenario in which the right-handed neutrinos are composite would naturally explain why these fermions are comparable or lighter than the compositeness scale Λ, assumed to be $O(\mathrm{TeV})$. Since their interactions with the Higgs boson through the dimension 5 operator (8) are not small, the Higgs boson would be composite as well (but presumed to be light).

These new decay channels of the Higgs boson will be the dominant decay modes if the right-handed neutrinos are also lighter than the electroweak gauge bosons, and if the coefficient of the higher dimension operator (8) is not too small. As discussed in Section 4, in the minimal flavor violation framework the predicted size of this operator depends on the choice of approximate flavor symmetries of the right-handed neutrinos. It may be $O(1)$ or $O(m_R/\Lambda)$.

In the former situation the new decays dominate over Higgs boson decays to bottom quarks for scales $\Lambda \lesssim 10-20$ TeV, although only scales $\Lambda \simeq 1-10$ TeV are technically natural. This case however presents a challenge to model building, since the operator (8) breaks the chirality of the right-handed neutrinos. Although it may be technically natural for the right-handed neutrinos to be much lighter than the scale Λ (see Section 3), one might expect that any theory which generates a large coefficient for this operator to also generate Majorana masses $m_R \sim O(\Lambda)$.

In the case where the coefficient of (8) is $O(m_R/\Lambda)$ the new decays can still dominate over decays to bottom quarks provided that the scale $\Lambda \simeq O(1\ \mathrm{TeV})$. For larger values of Λ these decays are subdominant but have sizable branching fractions up to $\Lambda \simeq O(10\mathrm{TeV})$. This situation might be more amendable to model building. For here an approximate $SU(3)$ symmetry is protecting the mass of the right-handed neutrinos.

In either case though one needs to understand why the right-handed neutrinos are parametrically lighter than Λ. It would be extremely interesting to find non-QCD-type theories

of strong dynamics where fermions with masses parametrically lighter than the scale of strong dynamics occur. Or using the AdS/CFT correspondence [8], to find a Randall-Sundrum type model [7] that engineers this outcome. The attitude adopted here has been to assume that such an accident or feature can occur and to explore the consequences.

Assuming that these theoretical concerns can be naturally addresseed, the Higgs boson physics is quite rich. To summarize, in the new process the Higgs boson decays through a cascade into a six-body or four-body final state depending on the masses of the right-handed neutrinos. First, it promptly decays into a pair of right-handed neutrinos, which have a macroscopic decay length anywhere from $O(100\mu m - 10m)$ depending on the parameters of the Majorana and Dirac neutrino masses. If one or two active neutrinos are very light, then the decay lengths could be larger. Decays occurring in the detector appear as a pair of displaced vertices. For most of the time each secondary vertex produces a quark pair and a charged lepton, dramatically violating lepton number. For a smaller fraction of the time a secondary vertex produces a pair of charged leptons or a pair of quarks, each accompanied with missing energy. From studying these decays one learns more about neutrinos and the Higgs boson, even if these channels should not form the dominant decay mode of the Higgs boson. The experimental constraints on this scenario from existing colliders and its discovery potential at the LHC will be described elsewhere [19] [24].

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